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Optimum design of laminated composite plates with cutouts undergoing large amplitude oscillations

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Abstract—The present investigation concerns minimum weight design of composite plates in the presence of cutouts undergoing large amplitude oscillations. The Ritz finite element model using a nine-noded C^0 continuity, isoparametric element along with a higher order displacement theory which accounts for parabolic variation of transverse shear stresses is used to predict the dynamic behavior. The optimal value is found by using a Genetic Algorithm with tournament selection. Results have been obtained for various cutout geometries like square, rectangular, circular and elliptical in the large amplitude range.

Keywords: Laminated plates; cutouts; finite element analysis; large amplitude oscillations; optimum design; genetic algorithm.

1. INTRODUCTION

Cutouts are inevitable in structures. Cutouts in structural members like aircraft wings made up of composite laminates may result in a change in the dynamic characteristics. Their effects are likely to be quite high when the plate is undergoing large oscillations. Dynamic analysis of such structures is an important field, and the designer has to concentrate on these effects while working with these structures. Specifically in spacecraft or aircraft structures where thin skins are used, these undesirable vibrations may cause sudden failures due to resonance. Moreover, these structures are designed in an efficient manner both in terms of weight and performance. In practical applications, the assumption that the laminate vibrates at a small amplitude may not always be valid. The laminates that are designed based on this assumption may not be realistic in the large amplitude range due to geometric nonlinearity. It is necessary, therefore, to understand the effect of large

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amplitude vibrations in optimum design of laminates for critical applications. Many researchers have studied the nonlinear static and dynamic analyses for laminated composite plates. Reddy and Chao [1] presented a finite element analysis of the large-deflection theory (in Von-Karman's sense) including transverse shear, governing moderately thick laminated anisotropic composite plates. Putcha and Reddy [2] developed a refined mixed shear finite element for the nonlinear bending analysis of laminated plates. Nonlinear bending analysis of laminated plate with a higher-order theory and with a higher-order C^1 continuous refined finite element method for laminated beams and plates is given by Gajbir *et al.* [3–5]. Nonlinear forced and free vibration analysis of laminated composite plates with a higher-order theory with a higher-order C^1 continuous refined finite element is also reported by Gajbir *et al.* [6–8]. Chandrasekhara and Tenneti [9] carried out the nonlinear static and dynamic analyses of heated laminated plates using a shear flexible finite element. Their model accounts for large deflections of the plate and non-uniform distributions of temperature. A nine-noded isoparametric element is used to obtain the numerical solutions. Shi and Mei [10, 11] developed a time domain formulation for the large amplitude free vibration of plates. The procedure of deriving the nonlinear equations of motion are discussed and accurate frequency-maximum deflection relations are obtained for the fundamental and higher nonlinear modes.

In recent years many researchers have used the genetic algorithm for the optimum design of composite structures. Optimization of laminate stacking sequence for buckling with strain constraints by Nagendra *et al.* [12], and by the same authors the design of composite laminates for buckling and strength constraints [13], and design of blade-stiffened composite panels by genetic algorithm approach by Nagendra *et al.* [14] are a few other notable works in this field. Work on the optimum design of composite laminates for maximizing laminate strength and stiffness with fixed number of plies has been done by Callahan and Weeks [15]. Kogiso *et al.* [16] applied the genetic algorithm with memory for design of minimum thickness composite laminates subject to strength, buckling and ply contiguity conditions. Mahesh *et al.* [17] have employed the genetic algorithm for optimal design of turbine blades. A simplified design method is developed to determine the optimum combination of layer materials, orientations and their thicknesses. Lin and Hajela [18] in their paper describe the implementation of genetic search methods for the optimal design of structural systems with a mix of continuous, integer and discrete design variables. This is proposed as an alternative to the branch-and-bound technique that is used in conjunction with non-linear programming methods. Other notable works in optimization of composite laminates, panels and space structures are given in [19–24]. They looked into various aspects of the genetic algorithm and reported its advantages in optimizing composite laminates or structures with buckling and strength constraints. To the authors' knowledge, so far no attempt has been made to optimize composite laminated plates in the presence of cutouts with large amplitude oscillations. The present work is a step towards optimizing composite laminated plates under dynamic constraints undergoing large amplitude oscillations.

2. FORMULATION FOR LARGE AMPLITUDE FREE VIBRATION

The problem is formulated for a plate of thickness h composed of orthotropic layers of thickness h_i with fibers oriented at angles $\pm\theta$ as shown in the Fig. 1.

The higher-order displacement model which gives parabolic variation of shear stresses over the thickness of the laminate, used for the present analysis is given as [25]

$$\begin{aligned}
u(x, y, z, t) &= u_0(x, y, t) + f_1(z)\psi_x(x, y, t) + f_2(z)\theta_x(x, y, t), \\
v(x, y, z, t) &= v_0(x, y, t) + f_1(z)\psi_y(x, y, t) + f_2(z)\theta_y(x, y, t), \\
w(x, y, z, t) &= w_0(x, y, t),
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
f_1(z) &= C_1 z - C_2 z^3, \\
f_2(z) &= -C_4 z^3,
\end{aligned}$$

with $C_1 = 1$, and $C_2 = C_4 = 4/3h^2$; u, v and w are the displacements in the x, y and z directions; u_0, v_0 and w_0 are displacements of the middle plane of the laminate and θ_x, θ_y, ψ_x and ψ_y are the rotations.

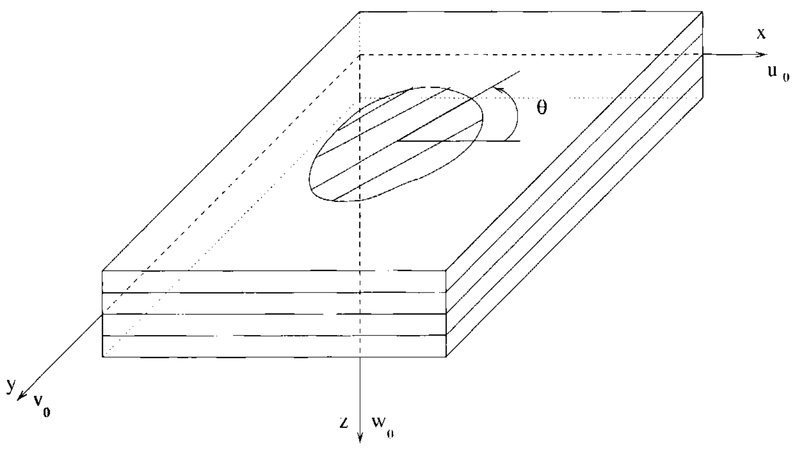

Figure 1. Laminated plate with coordinates and displacements.

From Green's strain vector, the nonlinear strain displacement relation given in [26] is

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} u_{,x} + \frac{1}{2}\left(u_{,x}^2 + \frac{1}{2}v_{,x}^2 + \frac{1}{2}w_{,x}^2\right) \\ v_{,y} + \frac{1}{2}\left(u_{,y}^2 + \frac{1}{2}v_{,y}^2 + \frac{1}{2}w_{,y}^2\right) \\ u_{,y} + v_{,x} + u_{,x}u_{,y} + v_{,x}v_{,y} + w_{,x}w_{,y} \\ u_{,z} + w_{,x} + u_{,x}u_{,z} + v_{,x}v_{,z} + w_{,x}w_{,z} \\ v_{,z} + w_{,y} + u_{,z}u_{,y} + v_{,z}v_{,y} + w_{,z}w_{,y} \end{Bmatrix}, \tag{2}$$

where

$$\{\varepsilon\} = \{\varepsilon\}^L + \{\varepsilon\}^{NL}, \tag{3}$$

in which the strain displacement relations corresponding to the model mentioned above are

$$\{\varepsilon\}^L = \begin{Bmatrix} \varepsilon_p^L \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b^L \\ \varepsilon_s \end{Bmatrix} + \begin{Bmatrix} 0 \\ z^2\varepsilon_s^* \end{Bmatrix} + \begin{Bmatrix} z^3\varepsilon^* \\ 0 \end{Bmatrix},$$

$$\{\varepsilon_p^L\} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix},$$

$$\{\varepsilon_b^L\} = C_1 \begin{Bmatrix} \psi_{x,x} \\ \psi_{y,y} \\ \psi_{x,y} + \psi_{y,x} \end{Bmatrix},$$

$$\{\varepsilon^*\} = -C_2 \begin{Bmatrix} \psi_{x,x} \\ \psi_{y,y} \\ \psi_{x,y} + \psi_{y,x} \end{Bmatrix} - C_4 \begin{Bmatrix} \theta_{x,x} \\ \theta_{y,y} \\ \theta_{x,y} + \theta_{y,x} \end{Bmatrix},$$

$$\{\varepsilon_s\} = C_1 \begin{Bmatrix} \psi_x \\ \psi_y \end{Bmatrix} + \begin{Bmatrix} w_{0,x} \\ w_{0,y} \end{Bmatrix},$$

$$\{\varepsilon_s^*\} = -3C_2 \begin{Bmatrix} \psi_x \\ \psi_y \end{Bmatrix} - 3C_4 \begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix}.$$

Assuming that the plate is moderately thick and strains are much smaller than the rotations, one can rewrite nonlinear components of equation (2) as

$$\{\varepsilon^{NL}\} = \begin{Bmatrix} \frac{1}{2}w_{,x}^2 \\ \frac{1}{2}w_{,y}^2 \\ w_{,x}w_{,y} \\ 0 \\ 0 \end{Bmatrix}.$$

This corresponds to the well known Von-Karman relationship for large displacements, where ( ), x and ( ),y represent derivatives with respect to x and y respectively.

The stress–strain relations for the kth lamina oriented at an arbitrary angle θ, with respect to the reference axis are

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{54} & \bar{Q}_{55} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}_k, \tag{4}$$

or

$$\{\sigma_i\} = [\bar{Q}_{ij}]\{\varepsilon_j\},$$

where $\bar{Q}_{ij}$s are the transformed elastic stiffness coefficients.

2.1. Energy equations

Strain energy of the plate is given by

$$U = \frac{1}{2}\int_v \varepsilon_i^T \sigma_i \, \mathrm{d}v. \tag{5}$$

The five strain component (plane stress condition) may be represented as ε_i and stress components as σ_i and for linear elastic constitutive matrix C_{ij} ($C_{ij} = \bar{Q}_{ij}$), the constitutive relations are given by

$$\sigma_i = C_{ij}\varepsilon_j. \tag{6}$$

Strain energy U can then be written as

$$\begin{aligned} U &= \frac{1}{2}\int\int\int \{\varepsilon\}^T C_{ij}\{\varepsilon\}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t \\ &= \frac{1}{2}\int\int\int \{\varepsilon^L + \varepsilon^{NL}\}^T C_{ij}\{\varepsilon^L + \varepsilon^{NL}\}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t \\ &= \frac{1}{2}\int\int\int \{C_{ij}(\varepsilon^L\varepsilon^L + 2\varepsilon^L\varepsilon^{NL} + \varepsilon^{NL}\varepsilon^{NL})\}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t. \end{aligned} \tag{7}$$

The strain component ε_i can be expressed as

$$\varepsilon_i = L_i^T d + \frac{1}{2} d^t H_i d, \tag{8}$$

in which L_i is a vector, H_i is a symmetric matrix and d is the vector of displacement gradients contributing to the strains. Using the procedure adopted by Rajasekaran and Murray [27] for isotropic plates and Ganapathi and Varadan [28] for composite laminates, the strain energy expression (membrane and bending) with higher order shear deformation theory for large amplitude free vibration can be written as,

$$U_{MB} = \frac{1}{2}\int\int d^T \left[\frac{1}{2}[NA] + \frac{1}{6}[NB] + \frac{1}{12}[NC]\right] d\,\mathrm{d}x\,\mathrm{d}y, \tag{9}$$

$$d^T = \langle u_{,x}\ u_{,y}\ v_{,x}\ v_{,y}\ w_{,x}\ w_{,y}\ \psi_{x,x}\ \psi_{y,y}\ (\psi_{y,x}, \psi_{x,y})\ \theta_{x,x}\ \theta_{y,y}\ (\theta_{y,x}, \theta_{x,y})\rangle.$$

The components of linear ($[NA]$) and nonlinear stiffness matrices ($[NB]$, $[NC]$) are given in the Appendix. Strain energy due to shear is expressed as,

$$U_S = \frac{1}{2}\int\int d_s^T [NS]\, d_s\,\mathrm{d}x\,\mathrm{d}y, \tag{10}$$

where

$$[NS] = \begin{bmatrix} [A_1] & [D_1] \\ [D_1] & [F_1] \end{bmatrix},$$

and

$$(A_{1_{ij}}, D_{1_{ij}}, F_{1_{ij}}) = \int_{-h/2}^{h/2} \bar{Q}_{ij}(1, z^2, z^4)\,\mathrm{d}z \quad \text{for } i, j = 4, 5.$$

Total strain energy for the laminate is given as,

$$U = U_{MB} + U_S. \tag{11}$$

The kinetic energy of the system can be expressed in terms of nodal degrees of freedom as follows:

$$T = \frac{1}{2}\int_A \left(\sum_{k=1}^{NL}\int_{z_{k-1}}^{z_k} \rho^{(k)}\dot{\bar{u}}^T\dot{\bar{u}}\right)\mathrm{d}A. \tag{12}$$

Here $\overline{u}$ is the global displacement vector and is given by

$$\{\overline{u}\} = \{u\ v\ w\}^T,$$

and

$$\{u\} = \left[\overline{N}\right]\{\delta\},$$

where

$$\left[\overline{N}\right] = \begin{bmatrix} 1\,0\,0 & f_1(z) & 0 & f_2(z) & 0 \\ 0\,1\,0 & 0 & f_1(z) & 0 & f_2(z) \\ 0\,0\,1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The kinetic energy T is therefore

$$T = \frac{1}{2}\int_A \left(\sum_{k=1}^{NL}\int_{z_{k-1}}^{z_k} \rho^{(k)}\dot{\delta}^T\left[\overline{N}\right]^T\left[\overline{N}\right]\dot{\delta}\right) \mathrm{d}A = \frac{1}{2}\int_A \dot{\delta}^T[m]\dot{\delta}\,\mathrm{d}A, \tag{13}$$

where $[m]$ is an inertia matrix, given as

$$[m] = \sum_{k=1}^{NL}\int_{z_{k-1}}^{z_k} \rho^{(k)}\dot{\delta}^T\left[\overline{N}\right]^T\left[\overline{N}\right]\dot{\delta}\mathrm{d}z = \begin{bmatrix} p & 0 & 0 & q_1 & 0 & q_2 & 0 \\ 0 & p & 0 & 0 & q_1 & 0 & q_2 \\ 0 & 0 & p & 0 & 0 & 0 & 0 \\ q_1 & 0 & 0 & I_1 & 0 & I_3 & 0 \\ 0 & q_1 & 0 & 0 & I_1 & 0 & I_3 \\ q_2 & 0 & 0 & I_3 & 0 & I_2 & 0 \\ 0 & q_2 & 0 & 0 & I_3 & 0 & I_2 \end{bmatrix},$$

with

$$(p, q_1, q_2, I_1, I_2, I_3) = \Big(\sum_{k=1}^{NL}\int_{z_{k-1}}^{z_k} \rho^k\big(1, f_1(z), f_2(2), f_1^2(z), f_2^2(z), [f_1(z), f_2(z)]\big)\mathrm{d}z.$$

2.2. Finite element model

In the present work, a C^0 nine-noded isoparametric quadrilateral finite element with 7 DOF per node (u, v, w, ψ_x, ψ_y, θ_x, θ_y) is employed. The full plate is idealized by an eight element mesh for finding the frequencies in the large amplitude range as given in Fig. 2. Reduced integration is used to evaluate the transverse shear stress while full integration is carried out for bending and stretching. Lagrangian shape functions are used to interpolate the generalized displacements within an element.

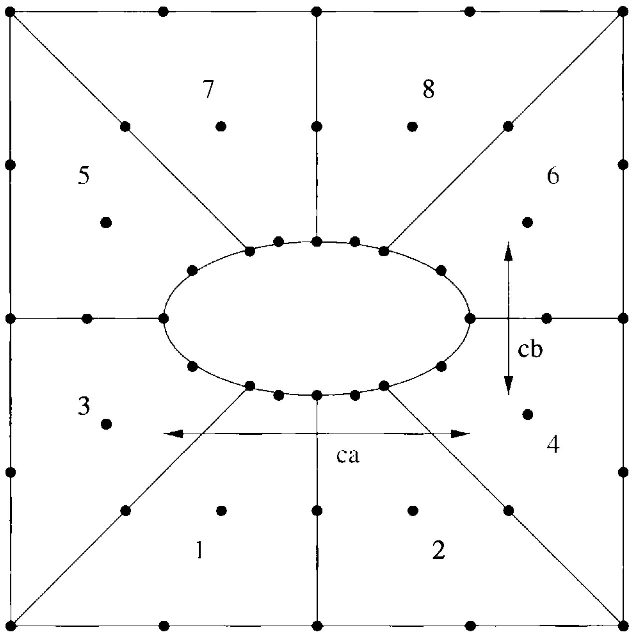

Figure 2. 8-element mesh of a laminated plate with elliptical cutout.

The generalized displacements within the element in terms of nodal displacements can be expressed as

$$\{\delta\}^e = \sum_{i=1}^{9} [N_i^e]\{q\}^e.$$

The displacement gradients can be related to the nodal displacements in the finite element modeling as

$$[d_{b_i}] = \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 & 0 & 0 \\ N_{i,y} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_{i,x} & 0 & 0 & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_{i,x} & 0 & 0 & 0 & 0 \\ 0 & 0 & N_{i,y} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_1 N_{i,x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_1 N_{i,y} & 0 & 0 \\ 0 & 0 & 0 & C_1 N_{i,y} & C_1 N_{i,x} & 0 & 0 \\ 0 & 0 & 0 & -C_2 N_{i,x} & 0 & -C_4 N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & -C_2 N_{i,y} & 0 & -C_4 N_{i,y} \\ 0 & 0 & 0 & -C_2 N_{i,y} & -C_2 N_{i,x} & -C_4 N_{i,y} & -C_4 N_{i,x} \end{bmatrix} \{q_{MB_i}\},$$

or

$$[d_b] = [B_{MB}]\{q_{MB}\},$$

$$d_{s_i} = \begin{bmatrix} 0 & 0 & N_{i,x} & 1 & 0 & 0 & 0 \\ 0 & 0 & N_{i,y} & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 & -3 \end{bmatrix} \{q_{S_i}\},$$

or

$$[d_S] = [B_S]\{q_S\}.$$

Using the normal procedure, the element stiffness matrices can be written as

$$[K_e] = \int_{-1}^{1}\int_{-1}^{1} B_{MB}{}^T \ [NA] \ B_{MB} \ \ J \,\mathrm{d}\psi \,\mathrm{d}\eta,$$

$$[K_{NL1_e}] = \int_{-1}^{1}\int_{-1}^{1} B_{MB}{}^T \ [NB] \ B_{MB} \ \ J \,\mathrm{d}\psi \,\mathrm{d}\eta,$$

$$[K_{NL2_e}] = \int_{-1}^{1}\int_{-1}^{1} B_{MB}{}^T \ [NC] \ B_{MB} \ \ J \,\mathrm{d}\psi \,\mathrm{d}\eta,$$

$$[K_{S_e}] = \int_{-1}^{1}\int_{-1}^{1} B_S{}^T \ [NS] \ B_S \ \ J \,\mathrm{d}\psi \,\mathrm{d}\eta. \tag{14}$$

If we now assemble these element matrices to get global matrices and vectors, the strain energy becomes

$$U = \frac{1}{2}\int\int d^T\left[\frac{1}{2}[K_{MB}] + \frac{1}{6}[K_{NL1}] + \frac{1}{12}[K_{NL2}] + \frac{1}{2}[K_s]\right] d \,\mathrm{d}x \,\mathrm{d}y. \tag{15}$$

The Lagrangian equation of motion for free vibration is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial U}{\partial q_i} = 0. \tag{16}$$

If we substitude the strain energy and kinetic energy expressions in equation (16), the governing equation for nonlinear eigenvalue problem is obtained as

$$[M]\{\ddot{\delta}\} + \left[[K_{MB}] + \frac{1}{2}[K_{NL1}] + \frac{1}{3}[K_{NL2}] + [K_s]\right]\{\delta\} = 0, \tag{17}$$

where $[K_{NL1}]$, $[K_{NL2}]$ are the nonlinear stiffness matrices corresponding to cubic and quadratic functions of the displacement vector. The above nonlinear eigenvalue problem is solved using the solution procedure for direct iteration method suggested in [28–31].

At the point of maximum amplitude

$$\{\ddot{\delta}\} = -\omega^2\{\delta\},$$

$$\{\dot{\delta}\} = 0.$$

Let,

$$[K^L] = [K_{MB}] + [K_S], \text{ (linear stiffness matrix)},$$

$$[K^{NL}] = \frac{1}{2}[K_{NL1}] + \frac{1}{3}[K_{NL2}], \text{ (nonlinear stiffness matrix)}.$$

The nonlinear eigenvalue problem now becomes,

$$\left[K^L + K^{NL}(\delta)\right]\{\delta\} - \omega^2[M]\{\delta\} = 0. \quad (18)$$

The solution of equation (18) is obtained using the direct iteration method as explained below:

Step 1. The linear eigenvalue problem is solved by setting the amplitude to zero in equation (18).

Step 2. The mode shape of the desired nonlinear mode is normalized with respect to the given amplitude at the point of maximum deflection.

Step 3. Using the normalized mode shape, the nonlinear stiffness matrix $[K_{NL}]$ is computed.

Step 4. The equations then are solved to obtain new eigenvalues and corresponding eigenvectors.

Step 5. Steps (2)–(4) are repeated until convergence is attained for $\{\delta\}_{max}$ and as well of ω^2 corresponding to this mode shape.

3. GENETIC ALGORITHM (GA)

A set of design alternatives called the initial population in the GA is allowed to reproduce and crossover among itself. Occasional random change in the strings which is similar to the changes in the genetic code in species to adapt itself in the environment in the long run is also allowed here with a small probability in the form of mutation. Since the search is not based on the gradient information, there is no requirement of continuity or convexity of the design space. A tournament selection scheme as discussed in [15], [32] is adopted here.

4. NUMERICAL EXAMPLES AND DISCUSSION

4.1. Boundary conditions

Simply supported : BC-1

$$u_0 = w_0 = \psi_y = \theta_y = 0 \quad \text{at } x = 0, a$$
$$v_0 = w_0 = \psi_x = \theta_x = 0 \quad \text{at } y = 0, b.$$

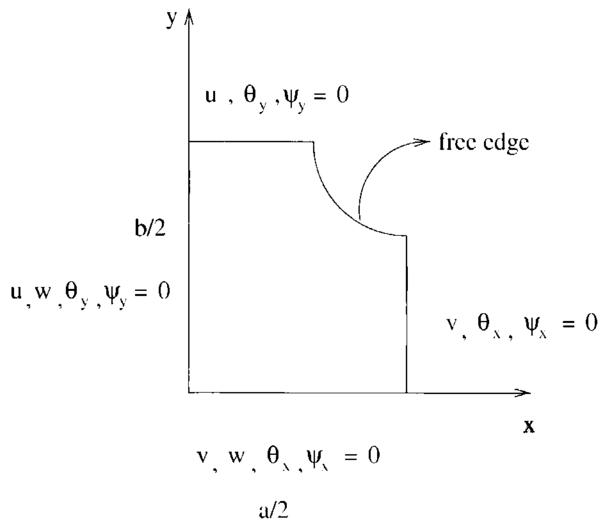

Figure 3. Quarter plate model with co-ordinates and end conditions.

Table 1.
Composite material properties

Material code	Composite material	E_1/E_2	G_{12}/E_2	G_{13}/E_2	G_{23}/E_2	ν_{12}	ρ kg/m^3
1	Graphite/epoxy	40	0.6	0.6	0.5	0.25	1500
2	Glass/epoxy	3.0	0.5	0.5	0.4	0.26	1800
3	Kevlar/epoxy	14.82	0.375	0.375	0.375	0.34	1460
4	Boron/epoxy	10	0.3	0.3	0.275	0.23	2000
5	Graphite/epoxy	15	0.429	0.429	0.357	0.25	1389

Clamped supported : BC-2

$$u_0 = v_0 = w_0 = \psi_x = \psi_y = \theta_x = \theta_y = 0 \quad \text{at } x = 0, a \text{ and } y = 0, b.$$

The material properties used are given in Table 1. A validation study is carried out with the proposed model for predicting the frequencies at large amplitudes. Table 2 gives the comparison of the present results using full-plate and quarter-plate models (see Fig. 3) with the results given in [33] for an isotropic square plate of $a/h = 10$ in the presence of a square cutout for various cutout ratios. From the table it is observed that the quarter-plate analysis gives fairly accurate results for isotropic plates. Maximum amplitude of vibration is taken as $A/h = 1.0$, where A is the maximum amplitude of vibration. Comparison of present results for angle ply and cross ply thin square laminates with the results given in Reddy [33] are presented in Table 3. given in [33]. Here again, it is observed that quarter-plate analysis is sufficient for an engineering accuracy.

Table 2.
Validation results on large amplitude vibration of isotropic plate with square cutout ($A/h = 1.0$, $\nu = 0.3$)

ca/a ratio	Frequency ratio								
	Present: full plate			Present: quarter plate			Reddy [33]: quarter plate		
	$a/h = 5.0$	$a/h = 10.0$	$a/h = 20.0$	$a/h = 5.0$	$a/h = 10.0$	$a/h = 20.0$	$a/h = 5.0$	$a/h = 10.0$	$a/h = 20.0$
0.2	1.5743	1.5270	1.5135	1.5743	1.5270	1.5135	1.5815	1.5121	1.4945
0.5	—	1.3500	1.3735	—	1.3651	1.3864	1.3653	1.3329	1.3248

Table 3.
Validation results on large amplitude vibration of two-layer angle ply square laminate with square cutout ($ca/a = 0.2, a/h = 1000.0$), Material: graphite/epoxy ($E_1/E_2 = 40.0, G_{12}/E_2 = G_{13}/E_2 = G_{23}/E_2 = 0.5\nu_{12} = 0.25$)

A/h ratio	Frequency ratio					
	Present: full plate		Present: quarter plate		Reddy [33]: quarter plate	
	[0°/90°]	[45°/−45°]	[0°/90°]	[45°/−45°]	[0°/90°]	[45°/−45°]
0.2	1.0428	—	1.0385	1.1509	1.0389	1.1636
0.4	1.1641	—	1.1443	1.3233	1.1499	1.3471

4.2. Minimimum weight design

Minimizing the weight of the plate in the presence of various cutouts with large amplitude of oscillations is discussed in this section. In this study, the probability of mutation (P_m) and the probability of crossover (P_c) varied from 0.0001 to 0.1, and 0.75 to 0.95, respectively. Best among the solutions obtained by four trial runs are given for each of the problems studied.

Two different optimization problems are considered:

Problem 1

$$\text{Minimize} \quad W$$
$$\text{subject to} \quad \omega_{1_{NL}} \geqslant \omega_s.$$

I. Weight of a four-layer symmetric orthotropic ($\theta = 0°$) simply supported square laminate length $a = 0.3$ m is minimized for a given lower bound value on fundamental frequency in the presence of various cutouts using the genetic algorithm. The material of the laminate is taken as graphite/epoxy (material code = 5 as in Table 1). The cutout parameters are taken as $ca/(a) = 0.2$, $ca/(cb) = 2.0$. (ca and cb are the length and width of the cutout, respectively). The specified limit on the fundamental frequency is $\omega_s \geqslant 1352.83$ Hz. The population in each generation is allowed to crossover and mutate with a probability of 0.75 to 0.95 and 0.0001 to 0.1, respectively. In the tournament selection procedure used here, the individuals have equal opportunity of getting selected and the individuals are competing twice in the tournament. The algorithm is run for specified number of generations to get the best solution. The range for the variable h (thickness) is taken as 0.001 to 0.006 and its string length is chosen to be 8. The results of computation are presented in Table 4. From this table it is observed that minimum weight is obtained in the case of a square laminate with rectangular cutout.

II. Minimum weight design of an eight-layer square, antisymmetric, cross-ply laminate simply supported on all edges is considered by treating ply thicknesses as variables with a specified limit on fundamental frequency discussed in this section. Here, the design variables are taken as discrete values in the range 1.0 mm to 7.75 mm with a step length of 0.25 mm. A five bit string

Table 4.
Minimum weight design of eight-layer symmetric orthotropic laminate in the presence of various cutouts with its edges simply supported on all sides ($ca/a = 0.2$, $ca/(cb) = 2.0$, $a = 0.3$, A/h=0.5)

Cutout	Optimized thickness (m)	Minimum weight (kg)	Frequency of optimized laminate $\omega_{1_{NL}}$ (Hz)	Constraint ω_s (Hz)
Square	0.002882 0.003333 0.003706 0.001451	2.587	1352.83	1352.83
Rectangular	0.002117 0.003882 0.002666 0.001745	2.485	1353.17	1352.83
Circular	0.005020 0.001157 0.003059 0.001863	2.578	1353.98	1352.83
Elliptical	0.001569 0.002078 0.003647 0.003098	2.506	1354.16	1352.83

Table 5.
Optimized thickness values for an sixteen-layer symmetric cross-ply simply supported laminate $[(0°/90°)_4]_s$ ($ca/a = 0.4$, $ca/(cb) = 2.0$, $a = 0.3$, $A/h = 0.3$, Material 5)

Shape of cutout	Optimized thickness (mm)	Obtained weight (kg)	Frequency $\omega_{1_{NL}}$ (Hz)	Constraint ω_s (Hz)
Elliptic	$[3.5/4.0/6.5/3.25/4.25/1.0/7.75/1.0]_s$	7.321	2865.75	2864.80
Rectangular	$[5.25/0.75/7.5/1.75/7.0/5.5/6.5/0.5]_s$	7.935	2868.40	2864.80

is used to represent the thickness in binary coding. The material properties used are low modulus graphite/epoxy (Material-5). In Table 5, the obtained optimum design values and constraint values are presented for elliptical and rectangular cutouts. It is observed that at the optimum the frequency constraint is active, hence the present design should be closer to the global optima.

Problem 2

Ply angles, ply materials and ply thicknesses are treated as variables in this study. Only discrete values are used for all design variables. Ply angles considered here are 0°, 90°, 45°, and −45°. Four different materials have been considered in this case, which are given in Table 1. A string length of 2 bits is used for each ply angle and ply materials. String length of the ply thicknesses are taken as 4 bits with minimum ply thickness and interval as 0.5mm.

The optimization problem is formulated as follows.

$$\begin{aligned} &\text{Minimize} && W \\ &\text{Subject to} && \omega_{1_{NL}} \geqslant \omega_s \\ &&& \theta \in 0^\circ, 90^\circ, 45^\circ, -45^\circ \\ &&& mc \in 2, 3, 4, 5 \text{ (Materials indicated in Table 1)} \end{aligned}$$

A thirty-layer symmetric laminate ($a = 1.0$ m) with clamped edges is taken for this study with elliptical cutout of size $ca/a = 0.3$, $ca/(cb) = 2.0$. The specified limit on the fundamental frequency is kept as $\omega_s = 1500.0$ Hz. The optimized thickness, constraint value at the optimum point and objective function values are given in the Table 6. Resulting minimum weight for the optimized solution is found to be 28.275 kg. Another minimum weight problem is studied for a lower bound value of $\omega_s = 1000.0$ Hz. Distribution of population in a typical run at initial and final generations is given in Fig. 4 for $\omega_s = 1000.0$ Hz.

Table 7 gives the optimized design for a twenty-layer clamped square symmetric laminate ($a = 0.8$ m) with square cutout $ca/a = 0.25$, oscillating at an amplitude ratio of $A/h = 0.8$. In the table, the solutions obtained at generation 1 and final optimized solution are presented. It is to be noted that though the solution one is also a feasible one, the weight obtained is much higher than the optimum. In Table 8 the minimum weight design of a twenty-layer symmetric square laminate

Table 6.
Minimum weight design values for a thirty-layer symmetric laminate with clamped edges in the presence of elliptical cutout ($ca/a = 0.3$, $ca/(cb) = 2.0$, $a = 1.0$)

Optimized design			Obtained weight (kg)	Frequency $\omega_{1_{NL}}$(Hz)	Constraint ω_s (Hz)
Ply thickness (mm)	Ply angle in degrees	Ply material			
$[0.5_2/1.5/0.5_8/2.5/0.5_3]_s$	$[90^\circ/0^\circ_2/-45^\circ/(0^\circ/45^\circ)_2/(90^\circ/45^\circ)_2/90^\circ_3]_s$	$[5_{12}/3/5/3]_s$	28.275	1511.33	1500.00
$[0.5_{15}]_s$	$[90^\circ/0^\circ_4/45^\circ/0^\circ/45^\circ/(90^\circ/45^\circ)_2/45^\circ/(-45^\circ)_2]_s$	$[5_{14}/3]_s$	20.167	1089.95	1000.00

($a = 0.8$ m) with circular cutout of $ca/a = 0.2$ is presented. The maximum amplitude of oscillation is taken at the same value as given in the above design. Two design problems are discussed in this table. For design I, the material for all the plies is varied. In the second design problem, the materials for eight inner layers are assigned to be kept as boron-epoxy (Material 4). The obtained minimum weights in both the designs are 37.194 kg and 48.012 kg, respectively. From the above tables, the power of the GA in approaching the optimum can be seen. It is clear from the table that once such assignment is done, the optimum weight for the given laminate may go up. The densities of boron and other materials are taken from Table 1.

Table 7.
Optimum design values for a twenty-layer symmetric clamped square laminate with square cutout ($a = 0.8$, $ca/a = 0.25$, $A/h = 0.8$)

Solution	Ply thickness (mm)	Ply angle (deg)	Ply material	Obtained weight kg	Frequeny $\omega_{1_{NL}}$	Constraint ω_s
At generation 1	[3.5/1.5/6.5/1.5/ 6.5/1.5 /0.5/5.5/3.0/1.5]$_s$	[90°/90°/45° /0°/45°/−45°/−45° 0°/90°/−45°]$_s$	[3/5/5/2/ 5/3 /3/5/4/4]	60.81	899.4	800.0
Optimized	[2.5/0.5/2.5/0.5/ 6.5/0.5 /0.5/5.5/2.5/0.5]$_s$	[90°/−45°/0° /45°/90°/−45°/−45° 0°/90°/−45°]$_s$	[5_{10}]$_s$	39.114	803.3	800.0

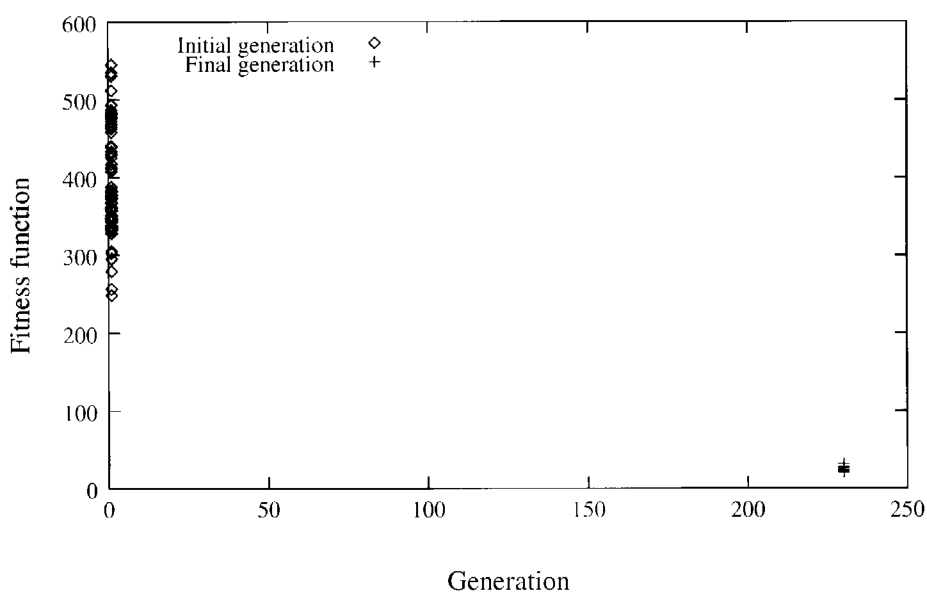

Figure 4. Distribution of population at initial and final generations in a typical run for the results given in Table 5.

Table 8.
Optimum design values for a ten-layer symmetric clamped square laminate with circular cutout ($a = 0.8$, $ca/a = 0.25$, $A/h = 0.8$)

	Optimized design			Obtained weight	Frequency $\omega_{1_{NL}}$ (Hz)	Constraint ω_s (Hz)
	Ply thickness (mm)	Ply angle (deg)	Ply material	(kg)		
I	[1.5/0.5/6.5/1.5/ 6.5/0.5 /0.5/0.5/2.5/1.5]$_s$	[90°/−45°/0° /−45°/90°/90°/0° 0°/90°/−e145°]$_s$	$[5_{10}]_s$	37.194	807.4	800.0
II	[2.5/0.5/6.5/1.5/ 2.5/0.5 /0.5/5.5/2.5/1.5]$_s$	[45°/0°/90° /90°/90°/90°/−45° 0°/90°/0°]$_s$	$[5_6/4_4^*]_s$	48.012	805.263	800.0

* Indicates that parameter(s) is/are not the variable.

5. CONCLUSION

For large amplitude vibration studies the present finite element model predicts the behavior quite satisfactorily. The presence of a cutout and its shape changes the behavior of the laminate in the large amplitude range. It is observed that the genetic algorithm is a versatile tool and can be used for study of complicated optimization problems involving design of composite laminates.

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APPENDIX

$$
[NA] = \begin{bmatrix}
A_{11} & A_{16} & A_{16} & A_{12} & 0 & 0 & B_{11} & B_{12} & B_{16} & E_{11} & E_{12} & E_{16} \\
 & A_{66} & A_{66} & A_{26} & 0 & 0 & B_{16} & B_{26} & B_{66} & E_{16} & E_{26} & E_{66} \\
 & & A_{66} & A_{26} & 0 & 0 & B_{16} & B_{26} & B_{66} & E_{16} & E_{26} & E_{66} \\
 & & & A_{22} & 0 & 0 & B_{12} & B_{22} & B_{26} & E_{12} & E_{22} & E_{26} \\
 & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & D_{11} & D_{12} & D_{16} & F_{11} & F_{12} & F_{16} \\
 & & & & & & & D_{22} & D_{26} & F_{21} & F_{22} & F_{26} \\
 & & & & & & & & D_{66} & F_{16} & F_{26} & F_{66} \\
 & & & & & & & & & H_{11} & H_{12} & H_{16} \\
 & & & & & & & & & & H_{22} & H_{26} \\
 & & & & & & & & & & & H_{66}
\end{bmatrix},
$$

$$
[NB_1] = \begin{bmatrix}
0 & 0 & 0 & 0 & A_{11}w_{,x} + A_{16}w_{,y} & A_{12}w_{,y} + A_{16}w_{,x} & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 0 & 0 & 0 & A_{16}w_{,x} + A_{66}w_{,y} & A_{26}w_{,y} + A_{66}w_{,x} & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & 0 & 0 & A_{16}w_{,x} + A_{66}w_{,y} & A_{26}w_{,y} + A_{66}w_{,x} & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & 0 & A_{12}w_{,x} + A_{26}w_{,y} & A_{22}w_{,y} + A_{26}w_{,x} & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & \begin{array}{c} A_{11}u_{,x} + A_{12}v_{,y} \\ +A_{16}(u_{,y} + v_{,x}) \end{array} & \begin{array}{c} A_{16}u_{,x} + A_{26}v_{,y} \\ +A_{66}(u_{,y} + v_{,x}) \end{array} & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & \begin{array}{c} A_{12}u_{,x} + A_{26}v_{,y} \\ +A_{26}(u_{,y} + v_{,x}) \end{array} & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & 0 & 0 & 0 & 0 \\
 & & & & & & & & & 0 & 0 & 0 \\
 & & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & & 0
\end{bmatrix},
$$